Homework 5 on Fracture, Thermal Conductivity

27-301, A. D. Rollett, Fall 2002

Due date: Sat., 12th Oct. ’02

Q1. [20 points]
Recall that a Case Study was given in lecture 7B on fracture toughness on the safe operating pressures of pressure vessels and materials choice (slide 15). Note that the use of English units is typical in the engineering world.
1a. The required operating stress for the pressure vessel is 180 ksi. From the four different grades of the steel discussed in the Case Study, select the grade that is best suited to meet the operating stress requirement. Show your quantitative reasoning for your choice (think along the lines of a safety calculation that has to be documented for Quality Control purposes).
1b. Alternatively, what would be the maximum allowable operating stress if the 220 ksi steel were to be used? Again, give a quantitative answer.
1c. Finally, compare the wall thicknesses of the pressure vessels made with the two different grades of steel.

Case Study: Failure Analysis of a Rocket Motor Case [from Courtney’s Mechanical Behavior of Materials]
A rocket motor case was made of a material that had a yield strength of 215 ksi (= 1485 MPa) and a $K_{IC}$ of 53 ksi(in)$^{1/2}$ (=58 MPa.m$^{3/2}$) and it failed at a stress of 150 ksi. Examination of the failed component showed that there was an elliptical surface crack with a depth of 0.039 inches (= 0.99 mm) and a length of 1.72 in (= 43.7 mm). Could this flaw have been responsible for the failure?

Answer:
The value of $f(c/a)$ for this flaw is 1.38. Rearranging the equation that relates fracture toughness to yield strength and operating stress, we obtain:

$$\sigma_{fracture} = \sqrt{\frac{f(c/a) 0.212 (\sigma_y/\sigma_s)^2}{1.20}[c]} K_{IC} = \sqrt{\frac{1.38 0.212 (\sigma_y/\sigma_s)^2}{1.20}[c]} K_{IC}$$

Now we estimate the fracture stress iteratively by substituting values of $K_{IC}$ and the crackdepth, $c$, (not the half-length!) and assume the operating stress value, $\sigma$, of 150 ksi, in order to estimate the RHS; then we compare the value on the RHS with the known fracture stress on the LHS. The answer turns out to be 156 ksi, which is not far off the actual fracture stress of 150 ksi. Substituting 156 ksi as the operating stress value, $\sigma$, into the RHS produces 156 ksi as the computed fracture stress. At this point the iteration has converged well enough for our purposes. The close agreement between the actual and the computed fracture
stresses suggests that the flaw was very likely to have been the cause of the failure.

Q2. [20 points]
Consider a pressure vessel made of a titanium alloy that is known to have an elliptical surface flaw with the same shape as the Case Study described above. Calculate the deepest crack of this type that can be tolerated (meaning that crack propagation is avoided) at an operating stress of 70% of the yield strength. The properties of the Ti alloy are $K_{IC} = 115$ MN.m$^{3/2}$ and a yield stress of 900 MPa.

Q3. [10 points]
Look up the thermal conductivity properties for graphite. You will find that graphites come in many different forms and grades so you will need to choose a grade and cite the source (just as if you were writing a paper). List both the density and the thermal conductivity of the grade that you choose. Then assume that you can make a graphite honeycomb of your chosen grade of graphite with a relative density of 0.3 and a perfect hexagonal structure (like a beehive). Estimate the thermal conductivity of your cellular graphite both parallel to the cell axis and perpendicular to it. Assume that the ratio of cell wall thickness to cell size is small (i.e. the walls are thin compared to the cell size).

Q4. [15 points]
Identify the symmetry elements that apply to composites that are made up of alternating layers of fibers laid at 0° and 90°. Write them in the form of (3x3 orthogonal) matrices [see lecture 4B for examples, and the homework on tensor properties].

Q5. [15 points]
Wood consists of fibers that are relatively stiff embedded in a matrix of softer material. If the (cellulose) fibers have a modulus of 30 GPa and the connecting material (hemicellulose and lignin) has a modulus of 0.5 GPa, what are the (Young's) moduli parallel and transverse to the grain? Assume that the volume fraction of fiber is 0.05 (=5%) and that the rest of the wood is composed of the connecting material (a gross approximation but good enough for the purposes of this question!), i.e. ignore the empty space inside the cells.

Q6. [10 points]
Explain how adding steel rods to concrete gives a useful composite material. Why are the steel reinforcing rods stretched (elastically) in tension before the concrete is allowed to set up around them?

Q7. [20 points]
Consider making fully dense composite materials with a ductile matrix and a brittle (strong) fiber. Assume that we are loading the composite parallel to the fiber axis and that the fibers are well bonded to the matrix so that the strain is equal in the two phases. The stress in the composite is given by the volume-weighted average:

$$\sigma_c = \sigma_m V_m + \sigma_f V_f$$
The stresses in the two phases are $\sigma_f = E_f \epsilon_f$, and $\sigma_m = E_m \epsilon_m$. This is only applicable until something changes inside the composite. Assume further that the fibers are brittle whereas the matrix work hardens up to its tensile strength. If the fibers fail at a stress that is less than the (ductile) failure strain of the matrix. Once the fibers fail, they no longer carry any load and so the matrix must carry all the load. This means that the load carrying capacity of the composite decreases with fiber additions.

At high enough volume fractions, however, the hardening in the matrix is exhausted before the failure strength of the fibers is reached. The matrix then fails at a stress, $\sigma^{\text{m}_f}$, which corresponds to the failure strain of the fibers. Under these conditions, the strength of the composite is an average of the strength of the fibers and the strength of the matrix at the failure strain of the fibers. The strength of the composite then increases with volume fraction of reinforcing fibers and is given by:

$$\sigma_c = \sigma^{\text{m}_f} V_m + \sigma_f V_f$$

[Note that this material will appear in the lectures on composites, lecture 9.]

The matrix has a modulus of 180 GPa and can be heat treated to have different properties. In condition A, it has a tensile strength of 800 MPa and in condition B is has a tensile strength of 600 MPa. The fibers have a strength of 1800 MPa and a failure strain of 0.3%.

(a) What is the stress in the matrix when the fibers fail, $\sigma^{\text{m}_f}$?

(b) Plot the composite strength as a function of fiber fraction for the two different heat treatment conditions of the matrix.

Q8. [15 points]

Why has the development of metal matrix composites focused on the low density metals for use as the matrix? Discuss this in terms of possible technological applications. Give three examples of metallic alloys that might be used as the matrix material with graphite fibers and compare their properties and suitability for use. In particular, try to predict what might happen if you heat the composite to a high fraction of the melting point of the matrix (as required, for instance, in heat treatment of the matrix).