1. Ferromagnetism

If the permeability of a material can be described by the following equation,

\[ B = aH + bH^2 + cH^3 \]

How would you make a linear approximation to the permeability?

[10] Very simple: just drop the second two terms on the RHS:

\[ B = aH \]

2. From the graph on slide 9 (Chen’s fig. 3.2), estimate the amount of energy lost in each cycle as depicted for the larger loop (max. field of ~2.8Oe). Express your answer in SI units. A table of conversion factors is provided at the end of the homework.

[20] The area enclosed of the right hand side is (very) approximately 12 kGOe. Therefore the total area is 24 kGOe. To convert from G to A.m\(^{-1}\) we multiply by 1,000. TO convert from Oe to A.m\(^{-1}\) we multiply by 1,000/4\(\pi\). To convert from (A.m\(^{-1}\))( A.m\(^{-1}\)) to J.m\(^{-3}\) we multiply by \(\mu_0 = 4\pi \times 10^{-7}\).

So, the energy lost per cycle = 24,000 * 1,000 * 1,000/4\(\pi\) * 4\(\pi\) * 10\(^{-7}\) = 2,400 J.m\(^{-3}\) per cycle.

[ equivalent to 24,000/10,000 * 1000/4\(\pi\) = 191 T.A.m\(^{-1}\)]

It is interesting to continue the calculation and estimate however crudely the "core loss", as expressed in W.kg\(^{-1}\) at 60Hz, by dividing by the density of iron and multiplying by the frequency.

Core Loss = 2,400 / 8,000 * 60 = 18 W.kg\(^{-1}\). This is rather high compared to the best transformer steels (also Fe-3Si) available today.

3. As you make it harder to move domain walls (e.g. by putting in more second phase particles), how would you expect the hysteresis loop to change? Sketch the “before” and “after” loops. Will the hysteresis loss increase or decrease as a result of the change? Which material would you prefer to use for a transformer?


4. In a strongly Goss-oriented material with a cube direction parallel to the rolling direction and a (110) face exposed on the rolling plane (see diagram below), what sort of domain walls would you expect to see on the surface (with no external field applied)? Would they be 90° or 180° walls, for example? What would you expect to observe at high field (the answer is much simpler!)?

[10] At low field, one would see a domain structure that closes off magnetization from exposing free poles at any surface. At high fields, only one domain exists: therefore no domain walls.
2. Symmetry

1. Consider a second order tensor property, $F$, in a cubic material; recall that a second order tensor relates vector quantities. Determine the restrictions that the symmetry operator that is a 90° rotation about the [001] axis places on the coefficients of the tensor $F$, i.e. write out the coefficients of the tensor (which ones are zero, which are equal to another coefficient). Hint: compare $F$ with $OFO^T$, and equate coefficients.

Now consider the effect of the symmetry operator that is a 90° rotation about the [100] axis. How many independent (non-zero) coefficients remain? You should be able to show with these two symmetry operations that the number of independent coefficients is only 1 in this case. (See p23 in Nye's book, for example.) Remember that the property must be the same after a symmetry rotation which means that you can equate the original property matrix and the rotated property matrix coefficient by coefficient, $F'_{ij} = F_{ij}$. I highly recommend using a program such as Mathematica (see below) or Maple that allows you to do matrix algebra automatically.

[20] In the Mathematica tableau below, notice that $mP$ is the original matrix (tensor) and $mP1$, $mP2$, etc. are the transformed matrices (tensors), each of which must be identical - component by component - with the original $mP$. The other matrices are rotation matrices that represent the symmetry operations.
\[
\text{mox90} = \begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 1 \\
0 & -1 & 0
\end{bmatrix}; \quad \text{MatrixForm[mox90]}
\]

\[
\text{MP} = \begin{bmatrix}
p11 & p12 & p13 \\
p21 & p22 & p23 \\
p31 & p32 & p33
\end{bmatrix}; \quad \text{MatrixForm[MP]}
\]

\[
\text{mox90t} = \text{Transpose[mox90]}; \quad \text{MatrixForm[mox90t]}
\]

\[
\text{mp2 = mox90 . MP . mox90t}; \quad \text{MatrixForm[mp2]}
\]

\[
\text{my90 = \{\{0, 0, 1\}, \{0, 1, 0\}, \{-1, 0, 0\}\};}
\quad \text{MatrixForm[my90]}
\]

\[
\text{my90t = Transpose[my90]}; \quad \text{MatrixForm[my90t]}
\]

\[
\text{mp3 = mox90 . MP . mox90t}; \quad \text{MatrixForm[mp3]}
\]

\[
\text{moz90 = \{\{0, 1, 0\}, \{-1, 0, 0\}, \{0, 0, 1\}\};}
\quad \text{MatrixForm[moz90]}
\]

General::spell :
 Possible spelling error: new symbol name "mox90" is similar to existing symbols \{mox90, my90\}.

\[
\text{moz90t = Transpose[moz90]}; \quad \text{MatrixForm[moz90t]}
\]

General::spell :
 Possible spelling error: new symbol name "mox90t" is similar to existing symbols \{mox90t, my90t, moz90\}.

\[
\text{mp4 = mox90 . MP . mox90t}; \quad \text{MatrixForm[mp4]}
\]

\[
\begin{bmatrix}
p22 & -p21 & p23 \\
-p12 & p11 & -p13 \\
p32 & -p31 & p33
\end{bmatrix}
\]
B2. [10 points] (a) The thermal conductivity of a material has been measured in single crystals of a monoclinic material (such that the 2-fold axis is aligned with the b-axis). When a temperature gradient of 10,000 K/m is applied along the a-axis, a heat flow of 100 Watts/m² is measured parallel to [100] face and 15 Watts/m² is measured across the (001) face. Which two entries in the thermal conductivity tensor have you measured, and what are their values?
You have measured $k_{11} = 10$ mW/(mK) and $k_{12} = 1.5$ mW/(mK).

(b) Comment on how you might design such an experiment. Hint: a sketch would be useful.
The key to doing this experiment is the arrangement of platens to contact the specimen that can be heated and/or cooled in order to control the heat flow and be able to measure it (e.g. with thermocouples in the cooling water lines). Be careful about the fact that $k_{12}$ is an off-diagonal term so the measurement of heat flow has to be performed perpendicular to the applied temperature gradient. This makes the experiment more challenging than just putting a specimen between two platens and measuring heat flow in different crystal directions parallel to the temperature gradient.

B3 (a). Repeat the sample problem given in the slides (problem 2.11 from Courtney’s book on Mechanical Behavior) in which you calculate and plot the variation in elastic modulus between [100] and [110] but using the properties of molybdenum (Mo) which has anisotropic elastic properties as follows. $C_{11} = 460$ GPa, $C_{12} = 176$ GPa, $C_{44} = 110$ GPa. You will need to convert the stiffness coefficients to compliance coefficients to that you can calculate $E_{100}$ and $E_{111}$. You can check your results by computing the modulus directly from the compliance coefficients (slide 49 of 301.L4B.11Sep02.tensors.ppt).

[25] The moduli for Mo from the slides are $C_{11} = 46$, $C_{12} = 17.6$ & $C_{44} = 11$ [10GPa]. Using the formulae given,

$S_{11} = (C_{11}+C_{12})/[(C_{11}-C_{12})(C_{11}+2C_{12})] = (46+17.6)/(46-17.6)(46+2*17.6) = (63.6)/(28.4)(81.2) = 0.02758$

$S_{12} = -C_{12}/[(C_{11}-C_{12})(C_{11}+2C_{12})] = -17.6/[(46-17.6)(46+2*17.6)] = -17.6/(28.4)(81.2) = -0.007632$

$S_{44} = 1/C_{44} = 0.09091$

Check: Zener’s anisotropy ratio $A = 2(S_{11}-S_{12})/S_{44} = 2(C_{11}-C_{12})/C_{44}$

$= 2(0.02758+0.007632)/0.09091 = 2*11/(46-17.6) = 0.775 = 0.775$ (reciprocal of Courtney’s value)

$E_{100} = 1/S_{11} = 1/0.02758 = 362.6$ [GPa]

$1/E_{111} = 2\left\{S_{11} S_{12} / S_{44}\right\}\{[\hat{q}^2]+[\hat{q}_2^2]+[\hat{q}_3^2]\}$ where $[\hat{q}] = 1/\sqrt{3}$,

So, $1/E_{111} = 0.02758 - 2(0.02758+0.007632-0.09091/2)(1/9+1/9+1/9) = 0.03441$

Or, $E_{111} = 290.6$ [GPa]

Similarly, for [110] where $[\hat{q}_1] = [\hat{q}_2] = 1/\sqrt{2}$, $[\hat{q}_3] = 0$,

$1/E_{110} = 0.02758 - 2(0.02758+0.007632-0.09091/2)(1/4+0+0) = 0.0327015$

Or, $E_{110} = 305.8$ [GPa].

Thus the plot is very similar to that in the example except that it goes down from [100] to [110] instead of rising. Note that Mo is stiffer in the <100> direction, which is a contrast to most cubic metals.

B3 (b). In a similar exercise, calculate and plot the variation in elastic (Young’s) modulus in copper for directions between [110] and [001] that are perpendicular to [1-10] (or, if you prefer, that lie in the plane (1-10)). This will show you the full range of modulus possible in a cubic material because the zone of <110> includes both <100> and <111>.

[15] Keep in mind in this problem that, although the direction cosines for the various directions between [110] and [001] all have $[\hat{q}] = [\hat{q}]$, then $[\hat{q}]$ is also varying and you must make a unit vector out of each combination. That is to say, start with $[\hat{q}] = [\hat{q}]$, and $[\hat{q}] = 0$ to represent [110] and end with $[\hat{q}] = [\hat{q}] = 0$, $[\hat{q}] = 1$, and normalizing with $\sqrt{[\hat{q}]^2+[\hat{r}]^2+[\hat{q}]^2}$. The angle, $[\hat{q}]$, is given by $\cos([\hat{q}])$. Other than that detail, you can apply the same formula as in the example problem in the slides! This is taken from Courtney’s book.
(c). Iron (Fe) has elastic moduli $C_{11} = 237$ GPa, $C_{12} = 141$ GPa, $C_{44} = 116$ GPa. You have been told to manufacture an iron sheet such that its elastic stiffness (modulus) through the thickness is 260 GPa. When you roll iron into a sheet, it develops a texture such that $<111>$ directions are parallel to the rolling plane normal (i.e. $<111>$ directions point “vertically” out of the plane of the sheet. Given that you start with randomly oriented material, whose modulus is 209 GPa, and assuming that the effect of the rolling is to convert randomly oriented material to $<111>$ oriented material linearly with strain (see sketch below), to what strain must you roll it in order to match the specified modulus? Hint: calculate the stiffness along the 111 direction.
As in the first one of these elasticity problems, you need to convert the moduli from stiffnesses to compliances and then calculate the modulus in the [111] direction. A quick way to do this is to note that

\[
\frac{1}{E_{111}} = \frac{S_{11}}{100} = \frac{1}{E_{100}} = \frac{C_{11}}{C_{12}} \left( \frac{C_{11} + 2C_{12}}{C_{12}} \right) = \frac{23.7 + 14.1}{23.7 + 2 \times 14.1} = \frac{37.8}{9.6 \times 5.19} = 0.0759 \ [1/10 \text{ GPa}]
\]

\[
E_{111} = \frac{1}{E_{100}} = 2.15; \text{ therefore } \frac{1}{E_{111}} = 0.0759/2.15 = 0.03529
\]

\[
E_{111} = 283.4 \ \text{GPa}
\]

The ratio of random to <111> fiber that gives the desired modulus (through the thickness) is

\[
200\% \times \frac{260-209}{283.4-209} = 137 \% \text{ strain.}
\]

Table of ELECTRICITY and MAGNETISM Conversion Factors


Permeability of free space (SI): \(4\pi \times 10^{-7}\)

<table>
<thead>
<tr>
<th>To convert from</th>
<th>To</th>
<th>Multiply by</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abampere</td>
<td>Ampere (A)</td>
<td>10</td>
</tr>
<tr>
<td>abcoulomb</td>
<td>Coulomb (C)</td>
<td>10</td>
</tr>
<tr>
<td>abfarad</td>
<td>Farad (F)</td>
<td>1.0 \times 10^9</td>
</tr>
<tr>
<td>abhreyn</td>
<td>Henry (H)</td>
<td>1.0 \times 10^9</td>
</tr>
<tr>
<td>abmho</td>
<td>siemens (S)</td>
<td>1.0 \times 10^9</td>
</tr>
<tr>
<td>abohm</td>
<td>ohm (\Omega)</td>
<td>1.0 \times 10^9</td>
</tr>
<tr>
<td>abvolt</td>
<td>volt (V)</td>
<td>1.0 \times 10^8</td>
</tr>
<tr>
<td>ampere hour (A · h)</td>
<td>coulomb (C)</td>
<td>3.6 \times 10^{11}</td>
</tr>
<tr>
<td>biot (B)</td>
<td>ampere (A)</td>
<td>10</td>
</tr>
<tr>
<td>EMU of capacitance (abfarad)</td>
<td>farad (F)</td>
<td>1.0 \times 10^9</td>
</tr>
<tr>
<td>EMU of current (abampere)</td>
<td>ampere (A)</td>
<td>10</td>
</tr>
<tr>
<td>EMU of electric potential (abvolt)</td>
<td>volt (V)</td>
<td>1.0 \times 10^8</td>
</tr>
<tr>
<td>EMU of inductance (abhreyn)</td>
<td>henry (H)</td>
<td>1.0 \times 10^9</td>
</tr>
<tr>
<td>EMU of resistance (abohm)</td>
<td>ohm (\Omega)</td>
<td>1.0 \times 10^9</td>
</tr>
<tr>
<td>ESU of capacitance (statfarad)</td>
<td>farad (F)</td>
<td>1.112650 \times 10^{12}</td>
</tr>
<tr>
<td>ESU of current (statampere)</td>
<td>ampere (A)</td>
<td>3.335641 \times 10^{10}</td>
</tr>
<tr>
<td>ESU of electric potential (statvolt)</td>
<td>volt (V)</td>
<td>2.997925 \times 10^{12}</td>
</tr>
<tr>
<td>ESU of inductance (stathreyn)</td>
<td>henry (H)</td>
<td>8.987552 \times 10^{11}</td>
</tr>
<tr>
<td>Physical Quantity</td>
<td>Symbol</td>
<td>Conversion Factor</td>
</tr>
<tr>
<td>---------------------------</td>
<td>--------</td>
<td>-------------------</td>
</tr>
<tr>
<td>ESU of resistance (statohm)</td>
<td>ohm</td>
<td>$8.987552 \times 10^{11}$</td>
</tr>
<tr>
<td>Faraday (based on carbon 12)</td>
<td>coulomb(C)</td>
<td>$9.648531 \times 10^{14}$</td>
</tr>
<tr>
<td>Franklin (Fr)</td>
<td>coulomb(C)</td>
<td>$3.335641 \times 10^{10}$</td>
</tr>
<tr>
<td>Gamma (γ)</td>
<td>Tesla (T)</td>
<td>$1.0 \times 10^{9}$</td>
</tr>
<tr>
<td>Gauss (Gs, G)</td>
<td>Tesla (T)</td>
<td>$1.0 \times 10^{4}$</td>
</tr>
<tr>
<td>Gauss (Gs, G)</td>
<td>Ampere/meter (A·m⁻¹)</td>
<td>$1.0 \times 10^{4}$</td>
</tr>
<tr>
<td>Gilbert (Gi)</td>
<td>Ampere (A)</td>
<td>$7.957747 \times 10^{11}$</td>
</tr>
<tr>
<td>Maxwell (Mx)</td>
<td>Weber (Wb)</td>
<td>$1.0 \times 10^{8}$</td>
</tr>
<tr>
<td>Mho</td>
<td>Siemens (S)</td>
<td>1.0</td>
</tr>
<tr>
<td>Oersted (Oe)</td>
<td>Ampere per meter (A/m)</td>
<td>$7.957747 \times 10^{11}$</td>
</tr>
<tr>
<td>Ohm centimeter (Ω·cm)</td>
<td>Ohm meter (Ω·m)</td>
<td>$1.0 \times 10^{6}$</td>
</tr>
<tr>
<td>Ohm circular-mil per foot</td>
<td>Ohm meter (Ω·m)</td>
<td>$1.662426 \times 10^{9}$</td>
</tr>
<tr>
<td>Ohm circular-mil per foot</td>
<td>Ohm square millimeter/meter (Ω·mm²/m)</td>
<td>$1.662426 \times 10^{9}$</td>
</tr>
<tr>
<td>Statampere</td>
<td>Ampere (A)</td>
<td>$3.335641 \times 10^{10}$</td>
</tr>
<tr>
<td>Statcoulomb</td>
<td>Coulomb (C)</td>
<td>$3.335641 \times 10^{10}$</td>
</tr>
<tr>
<td>Statfarad</td>
<td>Farad (F)</td>
<td>$1.112650 \times 10^{12}$</td>
</tr>
<tr>
<td>Stathenry</td>
<td>Henry (H)</td>
<td>$8.987552 \times 10^{11}$</td>
</tr>
<tr>
<td>Statmho</td>
<td>Siemens (S)</td>
<td>$1.112650 \times 10^{12}$</td>
</tr>
<tr>
<td>Statohm</td>
<td>Ohm(Ω)</td>
<td>$8.987552 \times 10^{11}$</td>
</tr>
<tr>
<td>Statvolt</td>
<td>Volt (V)</td>
<td>$2.997925 \times 10^{10}$</td>
</tr>
<tr>
<td>Unit pole</td>
<td>Weber (Wb)</td>
<td>$1.256637 \times 10^{7}$</td>
</tr>
</tbody>
</table>