Microstructure Properties I

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Homework 2: Recrystallization: ANSWERS

Due: 14 Sep 02

1) a) Contrast the driving forces for recrystallization and for grain growth.
   b) What is the driving force (in Pascals) for grain growth in Nickel, if the grain boundary energy is 0.75 J.m^{-2} and the grain size is 10 µm?
   c) What is the driving force for recrystallization (in Pa) in Ni, given that the shear modulus is 76 GPa, the Burgers vector is 0.25 nm, and the dislocation density is 10^{14}.m^{-2}?

1/Answer  [10 points each]
   a) Grain growth is driven by reduction in grain boundary area. Recrystallization is driven by reduction in the energy stored during plastic deformation in the form of dislocation tangles.
   
   b) The driving force, $P$, is equal to the energy content per unit volume associated with grain boundaries. Approximating the grains as cubical boxes, the area per unit volume multiplied by the energy is given by,

   $$ P = \frac{g \times \text{area}}{\text{volume}} = 0.75 \left(10^{-5}\right)^2 / \left(10^{-5}\right)^3 = 7.5 \times 10^4 \text{J.m}^{-3} = 0.075 \text{MPa}. $$

   c) For recrystallization, $E \frac{Gb^2}{\pi}$, which we can estimate as

   $$ 76 \times 10^9 \times 0.25 \times 10^{-9} \times 10^{14} = 4.75 \times 10^{15} \text{J.m}^{-3} = 0.475 \text{MPa}. $$

2) Read sections 5, 5.1 and 5.2 on nucleation and 3.3 on boundaries (esp. p132) in Porter & Easterling. Make a qualitative argument (or quantitative, for extra credit) as to why nucleation in recrystallization must be heterogeneous.

2/Answer: [20 points]

As pointed out in the lecture, the critical nucleus size that would be predicted by classical nucleation theory is of the order of 1µm. This is a very large size as is appropriate to a small driving force (compared to the driving forces found for phase transformation). From classical nucleation theory, the critical free energy for nucleation is given by,

$$ \Delta G^* = \frac{16\pi}{3}(\Delta G' \Delta G_d)^2 = \frac{16\pi}{3}0.5^3 \left(2.10^6\right)^2 = 5.2.10^{13} \text{J}. $$

which we can compare with the available thermal energy at a typical recrystallization temperature for Ni, say 500°C, where $kT = 1.38 \times 10^{21} \times 773K = 1.1 \times 10^{20} \text{J}$. The ratio of the two energies is 48 million which means that the nucleation is essentially zero!

The argument for heterogeneous nucleation can be made quantitative by considering one of the mechanisms such as strain-induced-boundary-migration (SIBM) in which a
segment of prior high angle boundary moves in response to a difference in stored energy. If no stored energy (dislocations) exists on one side and the full amount on the other, then the boundary will experience the full driving force, which is more than sufficient to move it fast enough for recrystallization to occur, as question 3 below estimates.

3) How fast does the grain boundary in fig. 3.26 (in Porter & Easterling) move?
Assume that the dislocation density in the unrecrystallized region ahead of the boundary is \(10^{15}\) m\(^{-2}\). Estimate the mobility by inserting the diffusion coefficient for self-diffusion of Ni into the equation given in the slides. Estimate all other quantities such as the grain boundary energy. Calculate the mobility of a high angle boundary first, and then calculate how quickly it will move under the stored energy driving force. This question is based on slides of the third lecture on recrystallization. Data on diffusion can be found in Porter & Easterling in Ch. 2. You can other sources for diffusion coefficients but you must cite the source. Be aware that the tables list a pre-exponential factor and an activation energy, from which you must calculate a diffusion coefficient for the particular temperature of interest.

3/Answer: [20 points]
First, estimate the driving force as the product of the dislocation density and the energy per unit length of a dislocation.

\[
E = \frac{G b^2}{2} \times \square
\]

\(\square = 10^{15} \text{ m}^2\)

\(G = 76 \text{ Gpa}\)

\(b = 2.49 \text{ Å}\)

\[E = 2.36 \text{ MPa}\]

Next, estimate the mobility as the product of \(b^2/kT \times D = M\)

\[D = D_0 \exp(-Q/kT) = 190.10^{-6} \text{ m}^2\text{s}^{-1} \exp\{-279.7.10^3/8.31*698\} = 2.17.10^{-25} \text{ m}^2\text{s}^{-1}.
\]

\[M = 1.4 \times 10^{-24} \text{ m}^4/(\text{Js})^{-1}\]

Lastly, multiply the mobility by the stored energy to obtain the velocity estimate:

\[V = ME = 3.3.10^{-18} \text{ m.s}^{-1}\]

This is clearly a very small number – of order \(10^8\) µm/hr! One reason for our estimate being so low is that we used the bulk, lattice diffusivity. A more appropriate diffusion coefficient is the one for grain boundary diffusion. From Smithell’s Metals Handbook, a reasonable pair of values for g.b. diffusion (assuming a boundary width of \(5.10^{-10}\) m) is as follows.

\[D_{gb} = D_0 \exp(-Q/kT) = 4.4.10^{-5} \text{ m}^2\text{s}^{-1} \exp\{-40.10^3/8.31*698\} = 4.45.10^{-8} \text{ m}^2\text{s}^{-1}.
\]

This is clearly a much larger value than the estimate above. From this we obtain,

\[M = 2.86.10^{-7} \text{ m}^4/(\text{Js})^{-1}\]

\[V = ME = 0.68 \text{ m.s}^{-1}\]

This velocity is very high – much higher than one would ever observe in grain growth or recrystallization but it gives some idea of the upper limit to the rate of boundary motion.