Peter E. Hart

How the Hough Transform Was Invented

EDITOR'S INTRODUCTION

Our guest in this column is Dr. Peter Hart. Dr. Hart was born on 17 February 1941 in Brooklyn, New York where he attended public elementary and high schools. He received the B.E.E. degree from the Renesselaer Polytechnic Institute in 1962 and the M.S. and Ph.D. degrees from Stanford University in 1963 and 1966, respectively. He is married to educational writer Diane Hart and they have one daughter, Laura. Dr. Hart is currently the chair and founder of Ricoh Innovations, Inc. in Menlo Park, California.

If you have visited the Computer History Museum in Mountain View, California, you probably saw Shakey, the world's first mobile, intelligent robot. Dr. Hart was the head of the project within SRI International that developed Shakey. Besides Shakey and the topic of the column, which is by itself an important milestone in the computing era, Dr. Hart has had many achievements in his career, and in this limited space we will highlight just some of them.

Our guest's leadership skills and vision are well proven by the number of companies and international research centers he founded and/or led. At SRI International, where he served as the director of the Artificial Intelligence Center, Dr. Hart coinvented the A* algorithm for finding the shortest path through a graph. This is the basic algorithm used today in various Web services and GPS products to compute driving directions. Also, while at SRI International, he invented the modern form of the Hough transform and coauthored one of the most cited references in the field of computer science, *Pattern Classification and Scene Analysis*, which was in print for more than 25 years before being supplanted by a second edition. The textbook has been translated into four languages: Russian, Japanese, Chinese, and Korean.

On top of all these early-career achievements, in 1980 our guest founded the world's first corporate artificial intelligence research laboratory at Fairchild/Schlumberger. In 1983 he cofounded Syntelligence, Inc. and delivered commercial expert system solutions having multimillion-dollar licensing fees. Since 1991 he has been at Ricoh, where he has responsibility for creating new technology and business opportunities for the worldwide Ricoh Group. Moreover, Dr. Hart was the first non-Japanese person to serve as a corporate officer of Ricoh Company, Ltd.

All these leadership positions did not distract Dr. Hart from technical research and innovations. He holds 70 U.S. and foreign patents, the most recent of which was issued in August 2009. Dr. Hart has shaped technological advancements not only by his own contributions and research leadership positions but also through his service on numerous committees on technology strategy that advised the director of the National Science Foundation (NSF), the Undersecretary of Defense, the director of Central Intelligence, and the administrator of NASA. Dr. Hart is an IEEE Fellow, ACM Fellow, and AAAI Fellow. In 1998 he received the IEEE Information Society Golden Jubilee Award for work done with Prof. Thomas M. Cover establishing error bounds on the nearest-neighbor rule for pattern classification.

With all these achievements comes a pleasing personality and an interest in music and sports. If you look at Dr. Hart's profile on Facebook, you will see him in his biking clothes. He is humble and likes people to call him Peter. He is a source of inspiration to all those who work with him and "he challenges all of his colleagues with his intellect and by identifying issues that everybody else misses and continuously making amazing technical inventions," says Berna Erol at Ricoh Innovations Ltd. She adds, "Oftentimes we hear from our colleagues that whenever they encounter difficult situations they ask themselves 'what would Peter do'." For these coworkers, Peter is the big brother who cares deeply about his employees and guides them technically to success.

In this article, Dr. Hart shares with readers how the Hough transform was invented. You will be intrigued to learn how this standard item in the computer vision tool kit evolved almost by chance from a geometric insight to a theoretically elegant and computationally efficient procedure.

Ghassan AlRegib

The Hough transform, used to detect geometric features like straight lines in digital images, is likely one of the most widely used procedures in computer vision [1]–[3]. Although nobody tabulates the frequency with which any particular algorithm or technique is used in computer vision, we can get some idea of its popularity by noting that Google Scholar returns over 22,000 citations in response to the search term "Hough transform." This is several times larger than the number of citations found for other classical computer vision operators such as the Sobel operator [3] or the Canny edge detector [4].

Where did this transform come from? You may vaguely recall learning that it

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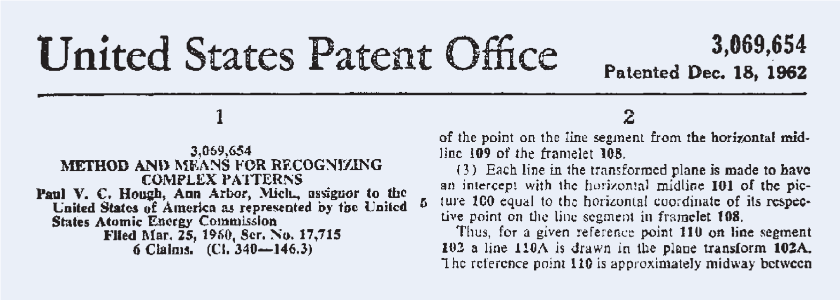
goes back to a 1962 patent by P.V.C. Hough [5], though I suspect very few readers have actually looked at that patent, the title page of which is shown as Figure 1. If you do, you may be surprised to find that the popular transform used today is not described there. Indeed, today's transform was not a single-step invention but instead took several steps that resulted in Hough's initial idea being combined with an idea from an obscure branch of late 19th century mathematics to produce the familiar sinusoidal transform.



United States Patent Office

3,069,654
Patented Dec. 18, 1962

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3,069,654
METHOD AND MEANS FOR RECOGNIZING COMPLEX PATTERNS
Paul V. C. Hough, Ann Arbor, Mich., assignor to the United States of America as represented by the United States Atomic Energy Commission
Filed Mar. 25, 1960, Ser. No. 17,715
6 Claims. (Cl. 340—146.3)

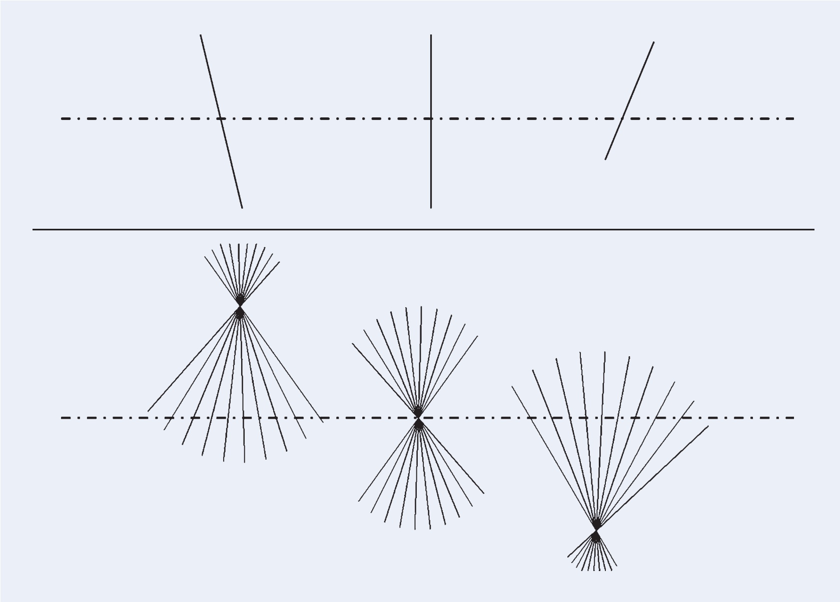
2

of the point on the line segment from the horizontal midline 109 of the framelet 108.

(3) Each line in the transformed plane is made to have an intercept with the horizontal midline 101 of the picture 100 equal to the horizontal coordinate of its respective point on the line segment in framelet 108.

Thus, for a given reference point 110 on line segment 102 a line 110A is drawn in the plane transform 102A. The reference point 110 is approximately midway between

[FIG1] From the title page of Paul V.C. Hough's patent.



[FIG2] Graphical description of the transform in the Hough patent. A point in the upper image space or "framelet" maps to a line in the lower "transformed space." Colinear points map to lines that intersect at a "knot." [The original figure in the patent has been redrawn and slightly simplified here for clarity.]

The previously untold history of how this came about illustrates how important advances sometime come from combining not-obviously-related ideas. The history perhaps also illustrates that the observation of Louis Pasteur, "Chance favors the prepared mind," remains as apt in the 20th and 21st centuries as it was in the 19th.

PAUL HOUGH'S TRANSFORM DEFINITION

The 1962 Hough patent is a marvel of brevity, being barely more than five pages long including figures and claims. More remarkable yet is that there are no algebraic equations defining any transform. Moreover, most of the invention disclosure, including two of the three figures, is devoted not to the transform but instead to a description of analog circuitry that uses components like difference amplifiers and sawtooth generators to implement the invention. This is indeed a succinct description of the transform idea.

The patent's sole figure describing the transform is shown in Figure 2 (redrawn to improve the clarity of the decades-old patent document), which is described as a "...geometric construction by hand..." A point in the upper half, or "framelet," maps to a straight line in the lower half or "transformed plane." The definition of the mapping is given in geometric rather than algebraic terms. The slope of a line in the transform space is defined as "...having an angle relative to the vertical whose tangent is proportional to the vertical displacement of the point...from the horizontal midline on the framelet." The position of a line in the transform space is defined as having "... an intercept with the horizontal midline... equal to the horizontal coordinate of its respective point...."

Hough implicitly recognized the problem that arises in the transform space when the horizontal intercept approaches infinity. He states "It is also necessary to scan each picture twice at right angles..."

Figure 2 shows three examples of how colinear points in the framelet map to intersecting lines in the transformed plane. These intersections are no accident: Hough obviously understood the theoretical properties of his geometrical construction for he states, "It is an exact theorem that, if a series of points in a framelet lie on a straight line, the corresponding lines in the plane transform [sic] intersect in a point, which we shall designate as a knot."

How Hough came upon the idea for this geometric, point-to-line transformation is a mystery even to Paul Hough himself. He had been seeking, without success, means to automate the tedious task of detecting and plotting the tracks of subatomic particles in bubble chamber

LONGEVITY

Paul V.C. Hough's most recent U.S. patent, #7,095,020, was issued in 2006, a remarkable 44 years after his transform patent was issued.

photographs. (The patent is assigned to the U.S. Atomic Energy Commission.) As related by Hough, while walking home from work one evening he had one of those inexplicable yet genuine "aha!" insights: Mapping a zero-dimensional point to a one-dimensional straight line—which by increasing the dimensionality seems to make the problem more complicated—actually led to a simple solution that could be implemented using analog electronic components of the day [6].

Regardless of how the idea came about, Hough's 1962 patent clearly disclosed a key idea that underlies the transform used today: colinear points in the image plane can be identified by mapping them into geometric constructions (for Hough, straight lines) that intersect in the transform space. But equally clearly, the geometric transform as described by Hough is hardly recognizable as the one that has been used by the computer vision community for decades; several more steps were needed to get there (see "Longevity").

AZRIEL ROSENFELD'S TRANSFORM DEFINITION

In 1969 Azriel Rosenfeld published his foundational book on computer vision, *Picture Processing by Computer*. In a single paragraph, seemingly almost as an afterthought at the end of a chapter, he presents an "interesting alternative scheme for detecting straight lines that makes use of a point-line transformation" [7]. He references the Hough patent and—for the first time, to the best of my knowledge—defines the transform algebraically in the form given by (1), where (x_i, y_i) are points in the image plane, and x and y are the axes of the transform plane:

$$y = y_i x + x_i. \qquad (1)$$

He points out that if a set of points $(x_i, y_i), i = 1, \ldots, n$, are colinear, then it is easily proved that the corresponding lines in the transform plane will all pass through a single point.

He also remarks parenthetically that if the points (x_i, y_i) are on a line that is nearly parallel to the x axis, then the lines become nearly parallel, so that their point of intersection recedes to infinity. Perhaps taking a hint from Hough's suggestion to "scan each picture twice at right angles," Rosenfeld recommends overcoming this difficulty by interchanging x_i and y_i in (1).

[THE 1962 HOUGH PATENT IS A MARVEL OF BREVITY, BEING BARELY MORE THAN FIVE PAGES LONG INCLUDING FIGURES AND CLAIMS.]

Rosenfeld made one additional recommendation. He notes, again parenthetically, that the transform space can be represented as an array of counters, so that "…the presence of many colinear 1's…" in the image plane will give rise to a high value in the array.

How Rosenfeld discovered the Hough patent cannot be determined at this late date. It is known that Hough and Rosenfeld were not personally acquainted [6]. Rosenfeld was certainly a prodigious scholar, and perhaps he simply found the patent on his own. But regardless of how he found it, we can say that Rosenfeld took several steps along the path that led from Hough's patent to the transform used today: He gave the first explicit algebraic form for the transform, he proposed a simple digital implementation of the transform space as an array of counters, and he introduced to the computer science and computer vision community an idea initially presented as an obscure—at least to that community—analog circuit-based patent.

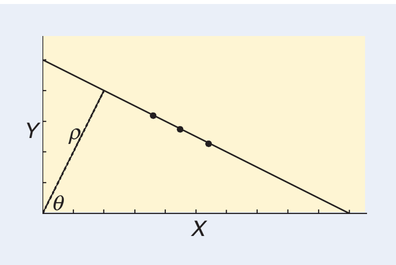
Azriel Rosenfeld was a towering figure in the history of computer image processing, with his ideas flowing across more than 600 papers and dozens of books. But I suspect that few are aware of the role he played in bringing one of the most important algorithms in computer vision to its present form.

A DETOUR THROUGH INTEGRAL GEOMETRY

In the early days of pattern recognition research, many alternative mathematical approaches were proposed that have long since been forgotten. One of these approaches was based on *integral geometry* [8], a branch of pure mathematics that studies the probability of random geometric events. A prototypical problem in integral geometry, first posed in the 18th century, is the *Buffon's needle* problem: Drop a needle on a floor made of planks and calculate the probability that the needle will lie across a crack. Other classical problems are to calculate the probability that a line intercepts a figure and to calculate the expected length of a random chord of a circle. During the 1960s, attempts were made by researchers in pattern recognition to use the statistics of random geometric events as a way to characterize properties of shapes in an image [9]. Some of this research came to my own attention during that period.

A key problem confronting mathematicians was how to formalize the notion of a "random" geometric event. Even an apparently simple event—tossing a line "at random" on a finite subset of a plane—can lead to paradoxes if a suitable probability space is not carefully defined.

Mathematicians resolved this problem by introducing an invariance requirement. If we require the results of random line-tossing calculations to be invariant to translation and rotation of geometrical figures in the plane, then it can be shown that there is only one parameterization of a line that can be



[FIG3] Using the normal parameterization of a straight line resolved the problem of "throwing a line at random" and also suggested a superior transform for computer vision purposes.

used. That is the radius-and-angle, or *normal*, parameterization defined algebraically by

$$y = -x\cos\theta/\sin\theta + \rho/\sin\theta \qquad (2)$$

and shown geometrically in Figure 3.

For integral geometers, the notion of throwing a line "at random" was now well defined. It meant sampling a uniform distribution on a rectangle in the $\rho - \theta$ space extending between 0 and say ρ_{max} in ρ and between 0 and 2π in θ.

For me, it suggested a solution to the problem of unbounded values of slopes and intercepts.

COMBINING IDEAS TO CREATE A NEW TRANSFORM

In the late 1960s I was working with my colleague Richard O. Duda and others to develop a vision system for SHAKEY, a mobile, intelligent robot created by the Artificial Intelligence Center of SRI International (SEE "SHAKEY"). SHAKEY lived in an indoor world composed of rooms populated with geometric solids like wedges and cubes. The vision systems of the day provided noisy edge detection as a starting point, and we were trying to use this noisy output to build a model of the local environment and to update the position of the robot from the identification of known landmarks.

I was investigating whether integral geometry approaches might help with this problem (they don't) and was at the same time studying Rosenfeld's *Image Processing by Computer* to see if I could pick up any ideas from that seminal volume. I was intrigued by Rosenfeld's brief description of the "point-line transformation" but was bothered by the awkward fact that the transform space was theoretically unbounded, requiring the inelegant axis-reversal trick previously mentioned.

The bounded values of ρ and θ of the normal form of a straight line, together with the invariance properties established by the integral geometers, suggested to me a new, and I thought more satisfactory, transform. From (2) we can

SHAKEY

SHAKEY is generally regarded as the world's first "mobile, intelligent robot." It has been inducted into the Robot Hall of Fame at Carnegie Mellon University and is on display at the Computer History Museum in Mountain View, California. A good overview of this historic project is given in [10].

map a point (x_i, y_i) in the image plane to the curve

$$\rho = x_i\cos\theta + y_i\sin\theta \qquad (3)$$

in a $\rho - \theta$ transform space.

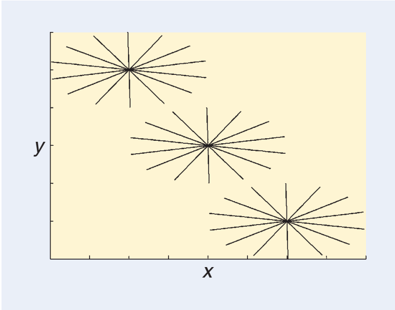
Figure 4 shows three points in an $x - y$ image plane and indicates that each point has an arbitrarily large number of lines passing through it. Is there a single, common line through all of them?

[HOUGH IMPLICITLY RECOGNIZED THE PROBLEM THAT ARISES IN THE TRANSFORM SPACE WHEN THE HORIZONTAL INTERCEPT APPROACHES INFINITY.]

Figure 5 shows three sinusoids in the $\rho - \theta$ transform space, with each sinusoid corresponding to a single point in the image plane. The values of ρ and θ at their intersection define the line in the $x - y$ plane that passes through the three points.

This new transform did not suffer from the theoretical and computational problems associated with mapping points to straight lines. Points in a finite image plane map to sinusoids in a finite transform space, and points along a line map to intersecting sinusoids regardless of line orientation or choice of coordinate axes.

Duda and I introduced the new transform to the research community in 1972 [11]. In that paper, we presented the first computational example illustrating its use, employing an array of counters as suggested by Rosenfeld



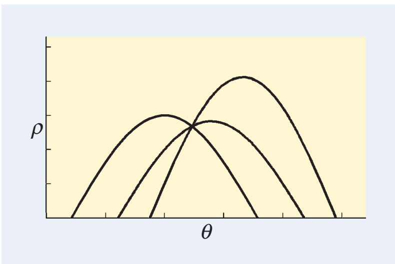
[FIG4] **Three points in an *x–y* image plane, with a family of lines passing through each of them. Is there a common line through all of them?**

to accumulate crossings of the sinusoids. We compared the computational complexity of the transform method with the complexity of an exhaustive analysis of all pairs of points in the image plane. A simple result was obtained showing that the comparative efficiencies of the transform approach and the exhaustive approach depended upon the number of points relative to the number of cells in the array of counters. We also introduced an extension of the transform to address the problem of detecting higher order analytic shapes like circles.

The transform as universally taught in textbooks and university courses today is the one described in the 1972 Duda and Hart paper.

LATER WORK AND CONCLUSIONS

I was initially surprised by what seemed to be an unenthusiastic reception by the research community of what I thought was a good idea, but my focus of attention started shifting



[FIG5] **Three sinusoids in a $\rho - \theta$ transform space corresponding to the three points of Figure 3. The intersection of the sinusoids corresponds to the line in the $x - y$ space passing through the three points.**

away from computer vision to other areas of artificial intelligence and to leadership responsibilities, and I thought no more about the transform. Many years later I was surprised yet again when I noticed that a 1988 survey article [12] cited 136 references. Research on the Hough transform and its variations continues to this day, and special issues of journals devoted to the transform occasionally appear, e.g., [13].

The breadth of application of the transform is illustrated by two very different recent examples. The first is in the same general problem area—particle tracking—that motivated Hough's patent. But we are many decades beyond bubble chambers, and the application uses a modified version of the transform to detect muon tracks in the large Hadron collider [14]. The second application is related to automotive safety: The transform is used in vision-based systems that detect when a vehicle is departing from a lane [15].

It is now more than 35 years since the modern form of the Hough transform was introduced by Duda and Hart, and it continues to be a standard item in the computer vision tool kit. But its early history shows how today's transform evolved almost by "by chance" from a geometric insight to a theoretically elegant and computationally efficient procedure.

ACKNOWLEDGMENT

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REFERENCES

[1] R. O. Duda and P. E. Hart, *Pattern Classification and Scene Analysis*. New York: Wiley-Interscience, 1973, pp. 335–337.

[2] R. Bradski and A. Kaehler, *Learning OpenCV*. Sebastopol, CA: O'Reilly, 2008, pp. 153–160.

[3] E. R. Davies, *Machine Vision: Theory, Algorithms, Practicalities*. San Francisco, CA: Elsevier, 2005, pp. 265–269.

[4] J. A. Canny, "A computational approach to edge detection," *IEEE Trans. Pattern Anal. Machine Intell.*, vol. 8, no. 6, pp. 679–714, 1986.

[5] P. V. C. Hough, "Method and means for recognizing complex patterns," U.S. Patent 3 069 654, Dec. 18, 1962.

[6] P. V. C. Hough, private communication, Mar. 13, 2009.

[7] A. Rosenfeld, *Picture Processing by Computer*. New York: Academic, 1969, pp. 335–336.

[8] H. Solomon, *Geometric Probability*. Philadelphia, PA: Society of Industrial Mathematics, 1987.

[9] A. B. J. Novikoff, "Integral geometry as a tool in pattern recognition," in *Principles of Self-Organization*, Von Foerster and Zopf, Eds. New York, NY: Pergamon, 1962, pp. 347–368.

[10] N. J. Nilsson, *The Quest for Artificial Intelligence: A History of Ideas and Achievements*. New York: Cambridge Univ. Press, 2010.

[11] R. O. Duda and P. E. Hart, "Use of the Hough transformation to detect lines and curves in pictures," *Commun. ACM*, vol. 15, no. 1, pp. 11–15, June 1972.

[12] J. Illingworth and J. Kitler, "A survey of the Hough transform," *Comput. Vis., Graph., Image Process. Arch.*, vol. 44, no. 1, pp. 87–116, 1988.

[13] *Real-Time Imaging*, vol. 6, no. 2, pp. 77–173, Apr. 2000.

[14] N. Amran and E. Etzion, "Hough transform track reconstruction in the cathode strip chambers in ATLAS," CERN-THESIS-2008-062, 2008.

[15] K. Mineta, K. Unoura, and T. Ikeda, "Development of a lane mark recognition system for lane keeping assist system," *Honda R & D Rev.*, vol. 12, no. 1, pp. 101–108, 2000.

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reader's CHOICE (continued from page 17)

TITLE, AUTHOR, PUBLICATION YEAR IEEE SPS CONFERENCES	ABSTRACT	RANK IN IEEE TOP 100 (JAN–JUNE 2009) JUN	MAY	APR	MAR	FEB	JAN	*N* TIMES IN TOP 100 SINCE JAN 2006
ENVIRONMENTAL ROBUSTNESS IN AUTOMATIC SPEECH RECOGNITION Acero, A.; Stern, R.M. *IEEE International Conference on Acoustics, Speech, and Signal Processing*, vol. 2, Apr. 1990, pp. 849–852	This paper proposes two novel methods that are based on additive corrections in the cepstral domain to deal with differences in noise level and spectral tilt between close-talking and desk-top microphones.		47					1
PERCEPTIBLE LEVEL LINES AND ISOPERIMETRIC RATIO Froment, J. *IEEE International Conference on Image Processing*, vol. 2, Sep. 2000, pp. 112–115	This paper introduces a simple criterion to select the most important level lines from the numerous set obtained with a topographic map.					72		1

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