27-750, A.D. Rollett
Due: 16th February, 2020.
Homework 4, Single Slip, and Multiple Slip Crystal Plasticity

Q1. [30 points] *Single Slip: Calculating Schmid Factors*
In a single crystal tensile test on Ni, the orientation of the crystal is given as (18.89°, 68.58°, 11.31°) in Bunge Euler angles. Assume that an axisymmetric tensile stress is applied along the sample 3 (Z) direction. Note that we apply stress boundary conditions, as discussed in class.

(a) Determine which crystal direction is parallel to the *tensile axis*. Give your answer in the form of (hkl)[uvw] and simplify the numbers to be single digit integers. You should be able to modify the spreadsheet or program that you wrote in earlier homeworks to produce an answer.

(b) Identify which combination of slip plane and direction is active, and calculate the Schmid factor (to 3 significant figures).

Note: Ni is fcc and can slip on any {111} plane in any <110> direction. You will have to find out which is the most highly stressed slip system, i.e. find the largest Schmid factor. This value of the Schmid factor is what you should use to determine the tensile yield stress because it determines which slip system will be activated first.

(c) Calculate the tensile yield stress (to 3 significant figures) based on a critical resolved shear stress that we will (arbitrarily) set at 100 MPa.

You must submit a copy of the table of results showing how you calculated your answer which must display the maximum Schmid factor and the Miller indices of the activated slip system.

Q2. [30 points] *Compute Slip Matrices; verify 384 non-singular cases out of 792*
Referring to slides 43 through 55 and #50 in particular, construct all the 5x5 slip matrices and determine which ones are non-singular. Verify that out of the ${}_{5}C_{12}$ combinations, only 384 are valid solutions, i.e., have a non-zero determinant.
Hints:
Clearly you have to start by making a list of all 12 slip systems in fcc metals based on {111}<110>. Although this problem is deceptively simple, the main programming challenge is to compute a list of all the possible combinations of five slip systems (as opposed to just the number). Fortunately the web has code fragments that will do this (in C++, Fortran etc.), and Matlab has a built-in function.
What to submit:
You should submit your code (e.g., python or Matlab script), as well as the list of 384 non-singular cases.

Q3. [40 points] *Bishop-Hill Model: Calculation of Taylor Factors for Multiple Slip*

Assume that slip occurs on {111} planes and in <110> directions. For each of the three different strain types, uniaxial tension parallel to the Z axis, plane strain in the +X & -Z, and simple shear in the XZ sense (see the matrices below), calculate the following quantities.
(a) the index of the active stress state (from the Bishop-Hill list);
(b) the Taylor factor (to at least 2 decimal places) for crystals with the following orientations;
(c) the active multi-slip stress state (as in the values of A, B, C, F, G, & H);
for crystals with the following orientations.

2.1 (0 0 1) [1 0 0]
2.2 (0 8 10) [1 0 0]
2.3 (-1 1 0) [1 1 1]
2.4 (2 1 3) [3 6 -4]
2.5 (1 1 2) [1 1 -1].

The strain tensors for the 3 different strain types are as follows.

Uniaxial tension // Z:

$$\begin{bmatrix} -0.5 & 0 & 0 \\ 0 & -0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Plane strain compression on +X and -Z:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Simple Shear on XZ:

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Question	Orientation	Taylor Factor for Uniaxial Tension //Z	Stress State
2.1a	(0 0 1) [1 0 0]	2.449	1.0, -1.0, 0.0, 0.0, 0.0, 0.0

2.2a	(0 8 10) [1 0 0]	3.585	0.0, 0.0, 0.0, 1.0, 0.0, 0.0
2.3a	(-1 1 0) [1 1 1]	3.674	(to be determined by you)

Answers are provided (above) for a few of the cases so that you can check your method. The meaning of the Miller indices is the conventional one discussed earlier in the class: (hkl) // sample-Z (=sample-3), [uvw] // sample-X (= sample-1).

The pseudo-code to obtain the Taylor factor is given in the lecture notes.

If you consider uniaxial tension for the three corners of the unit triangle (i.e. 001, 110 and 111), in which two corners is the inverse Schmid factor equal to the Taylor factor, and in which corner are they different?

Bishop & Hill stress states (Fortran style); factor of √6 *not* included:

```
       stress(1,1)=1.
       stress(1,2)=-1.
       stress(1,3)=0.
       stress(1,4)=0.
       stress(1,5)=0.
       stress(1,6)=0.
!   # 1
       stress(2,1)=0.
       stress(2,2)=1.
       stress(2,3)=-1.
       stress(2,4)=0.
       stress(2,5)=0.
       stress(2,6)=0.
!   # 2
       stress(3,1)=-1.
       stress(3,2)=0.
       stress(3,3)=1.
       stress(3,4)=0.
       stress(3,5)=0.
       stress(3,6)=0.
!   # 3
       stress(4,1)=0.
       stress(4,2)=0.
       stress(4,3)=0.
       stress(4,4)=1.
       stress(4,5)=0.
       stress(4,6)=0.
!   # 4
       stress(5,1)=0.
       stress(5,2)=0.
       stress(5,3)=0.
       stress(5,4)=0.
       stress(5,5)=1.
       stress(5,6)=0.
!   # 5
       stress(6,1)=0.
       stress(6,2)=0.
       stress(6,3)=0.
       stress(6,4)=0.
       stress(6,5)=0.
       stress(6,6)=1.
!   # 6
```

```
        stress(7,1)=0.5
        stress(7,2)=-1.
        stress(7,3)=0.5
        stress(7,4)=0.
        stress(7,5)=0.5
        stress(7,6)=0.
!   # 7
        stress(8,1)=0.5
        stress(8,2)=-1.
        stress(8,3)=0.5
        stress(8,4)=0.
        stress(8,5)=-0.5
        stress(8,6)=0.
!   # 8
        stress(9,1)=-1.
        stress(9,2)=0.5
        stress(9,3)=0.5
        stress(9,4)=0.5
        stress(9,5)=0.
        stress(9,6)=0.
!   # 9
        stress(10,1)=-1.
        stress(10,2)=0.5
        stress(10,3)=0.5
        stress(10,4)=-0.5
        stress(10,5)=0.
        stress(10,6)=0.
!   # 10
        stress(11,1)=0.5
        stress(11,2)=0.5
        stress(11,3)=-1.
        stress(11,4)=0.
        stress(11,5)=0.
        stress(11,6)=0.5
!   # 11
        stress(12,1)=0.5
        stress(12,2)=0.5
        stress(12,3)=-1.
        stress(12,4)=0.
        stress(12,5)=0.
        stress(12,6)=-0.5

        stress(13,1)=0.5
        stress(13,2)=0.
        stress(13,3)=-0.5
        stress(13,4)=0.5
        stress(13,5)=0.
        stress(13,6)=0.5

        stress(14,1)=0.5
        stress(14,2)=0.
        stress(14,3)=-.5
        stress(14,4)=-0.5
        stress(14,5)=0.
        stress(14,6)=0.5

        stress(15,1)=0.5
        stress(15,2)=0.
        stress(15,3)=-0.5
        stress(15,4)=0.5
        stress(15,5)=0.
        stress(15,6)=-0.5

        stress(16,1)=0.5
        stress(16,2)=0.
```

```
stress(16,3)=-0.5
stress(16,4)=-0.5
stress(16,5)=0.
stress(16,6)=-0.5

stress(17,1)=0.
stress(17,2)=-0.5
stress(17,3)=0.5
stress(17,4)=0.
stress(17,5)=0.5
stress(17,6)=0.5

stress(18,1)=0.
stress(18,2)=-0.5
stress(18,3)=0.5
stress(18,4)=0.
stress(18,5)=-0.5
stress(18,6)=0.5

stress(19,1)=0.
stress(19,2)=-0.5
stress(19,3)=0.5
stress(19,4)=0.
stress(19,5)=0.5
stress(19,6)=-0.5

stress(20,1)=0.
stress(20,2)=-0.5
stress(20,3)=0.5
stress(20,4)=0.
stress(20,5)=-0.5
stress(20,6)=-0.5

stress(21,1)=-0.5
stress(21,2)=0.5
stress(21,3)=0.
stress(21,4)=0.5
stress(21,5)=0.5
stress(21,6)=0.

stress(22,1)=-0.5
stress(22,2)=0.5
stress(22,3)=0.
stress(22,4)=-0.5
stress(22,5)=0.5
stress(22,6)=0.

stress(23,1)=-0.5
stress(23,2)=0.5
stress(23,3)=0.
stress(23,4)=0.5
stress(23,5)=-0.5
stress(23,6)=0.

stress(24,1)=-0.5
stress(24,2)=0.5
stress(24,3)=0.
stress(24,4)=-0.5
stress(24,5)=-0.5
stress(24,6)=0.

stress(25,1)=0.
stress(25,2)=0.
stress(25,3)=0.
stress(25,4)=0.5
```

```
stress(25,5)=0.5
stress(25,6)=-0.5

stress(26,1)=0.
stress(26,2)=0.
stress(26,3)=0.
stress(26,4)=0.5
stress(26,5)=-0.5
stress(26,6)=0.5

stress(27,1)=0.
stress(27,2)=0.
stress(27,3)=0.
stress(27,4)=-0.5
stress(27,5)=0.5
stress(27,6)=0.5

stress(28,1)=0.
stress(28,2)=0.
stress(28,3)=0.
stress(28,4)=0.5
stress(28,5)=0.5
stress(28,6)=0.5
```