Homework 3; due Saturday 8th Feb. '20
27-731
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100 points

Tensors, Rotations, Transformations, Anisotropic Elasticity

1. *Sample* to *crystal* axis transformations are very important in the context of texture and anisotropy. Being able to change from one to the other is therefore very important, particularly when dealing with tensors.

a) [5 points]. Calculate the transformation matrix α_{ij} from *sample* to *crystal* axes for the "cube orientation" with Euler angles $(\phi_1,\Phi,\phi_2) = (0°, 0°, 0°)$. How do the components of α_{ij} make physical sense?

b) [5 points] Calculate a transformation matrix from a set of Miller indices (hkl)[uvw] = (001)[100]

c) [5 points] Are your answers in parts a) and b) equivalent? Why or why not?

2. Tensors are defined as a quantity that obeys certain have transformation "rules". For example, a second ranked tensor T_{ij} transforms as: $T'_{ij} = \alpha_{im}\alpha_{jn}T_{mn}$. However, in general, we prefer to use "short hand" matrix notation of the form: $\mathbf{T'} = \boldsymbol{\alpha}\mathbf{T}\boldsymbol{\alpha}^{\mathbf{T}}$ (note **BOLD** refers to a matrix).

a) [10 points] Show the general expression for T'_{ij} for an arbitrary rotation θ about the [001] axis e.g. [θ@ [001]).

b) [5 points] For a θ=180-degree rotation about [001], what is **T'**? How does it compare to **T**? How do the components of our α matrix make physical sense in describing the rotation?

3. The Youngs Modulus E_{hkl} of a specific crystallographic plane (hkl) is given by:

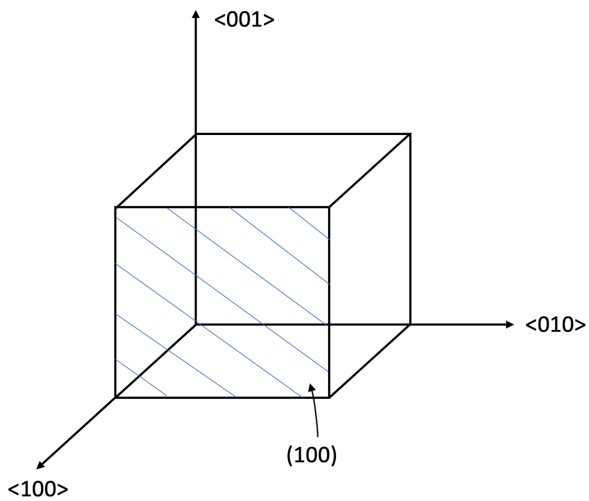
$$\frac{1}{E_{hkl}} = S_{11} - 2\left[(S_{11} - S_{12}) - \frac{1}{2}S_{44}\right](\alpha^2\beta^2 + \alpha^2\gamma^2 + \beta^2\gamma^2)$$

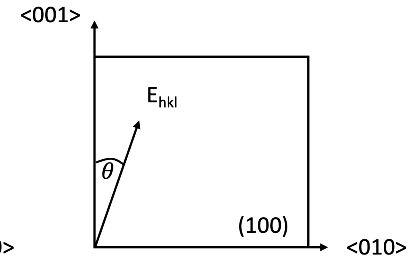
Where S_{ij} are components of the compliance tensor and α, β, and γ are the direction cosines of the principle directions in a cubic lattice. Direction cosines are given as follows: α = the cosine of the angle between (hkl) and [100]; β = the cosine of the angle between (hkl) and [010]; and γ is the cosine of the angle between (hkl) and [001].

a) [10 points] Which direction E_{111}, E_{100}, or E_{110} is the stiffest for an Fe-alloy where: S_{11} = 9.88×10^{-12} Pa^{-1} , S_{12}= -3.77×10^{-12}Pa^{-1}, and S_{44} = 7.22×10^{-12}Pa^{-2}?

Hint: A direction cosine can also be written as: $\frac{(hkl)\cdot[uvw]}{|(hkl)|\cdot|[uvw]|}$

b) [12 points] Within the (100) plane, show where the maximum E_{hkl} value(s) occur for an arbitrary vector making an angle with the [100] loading axis between 0 and 90 degrees. Which E_{hkl} value(s) are the least? Which are the largest E_{hkl} value(s)?





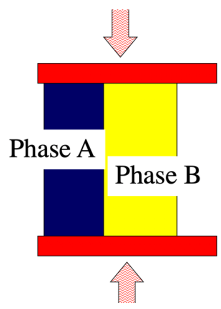
c) [3 points] Prove this material is either a: (1) *relatively* elastically isotropic, or (2) an anisotropic material.
Hint: we can convert $C=S^{-1}$ via the following relationships:

$$C_{11} = (S_{11}+S_{12})/\{(S_{11}-S_{12})(S_{11}+2S_{12})\} = 0.1913$$

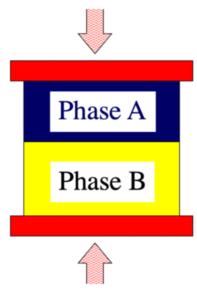
$$C_{12} = - S_{12}/\{(S_{11}- S_{12})(S_{11}+2S_{12})\}=0.118$$

$$C_{44} = 1/S_{44} = 0.138$$

4. [25] Plot the composite moduli as a function of volume fraction for a Cu-W material with properties: E_{copper} = 120 GPa and $E_{Tungsten}$ = 411 GPa for the "Isostress" and "Isostrain" cases. How do experimental results for Cu-W compare to these two bounds?



"Isostrain"



"Isostress"

Hint: recall the equations for composite modulus for Isostress and Isostrain cases:

$$E_{C,Voigt} = {V_f}^A E_A + {V_f}^B E_B$$

$$E_{C,Reuss} = \frac{E_A E_B}{{V_f}^A E_B + {V_f}^B E_A}$$

5. [20] EBSD data acquisition
(a) Describe in your own words how an accumulator diagram is used to apply the Hough transform to an EBSD diffraction image. Describe explicitly with an equation(s) how intensity values are transferred from the original image (diffraction pattern) to the accumulator image (diagram).
(b) Run the python code provided (*hough-python-v1.py*) and make any adjustments needed to obtain images like the ones below. This code ran on my 2010 Macbook with 10.13 and python3 (installed from python.org).
image.png and *accumulator.png*
(c) Modify the code so as to obtain the Hough transform of a band such as one sees in EBSD patterns, i.e., a line of finite width, and,
(d) the Hough transform of two intersecting bands. In both cases, describe what you see. You may find it helpful to look for the blog written by Stuart Wright about EBSD.