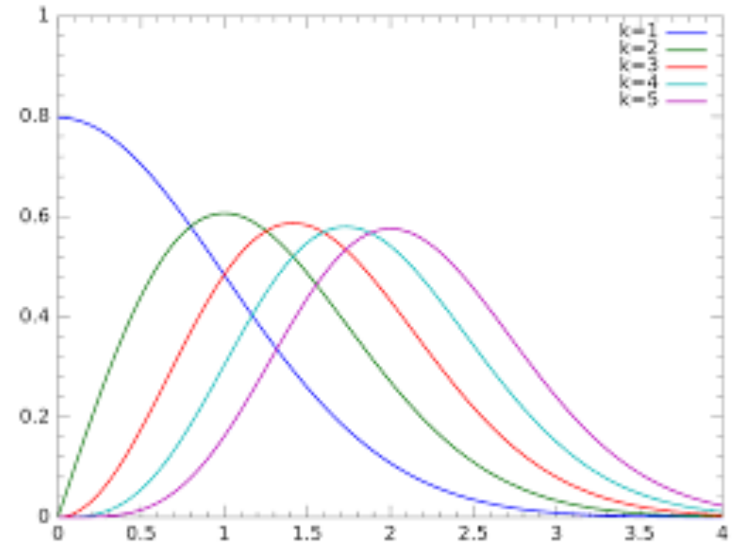
Statistics: χ^2 distribution

- Given a set of N mutually independent normal random variables Z_i with mean=0 and variance=1, the random variable $\sum_{i=1}^{N} Z_i^2$ has a χ^2 distribution with N degrees of freedom.
- The distribution is positive valued and skewed to the right.
- The sum of independent χ^2 variables is also a χ^2 random variable.
- Used to describe the sampling distribution of the sample variance s^2 for samples from a normal population. More to the point, we use it for likelihood ratio test statistics



In the plot, k is the number of variables.

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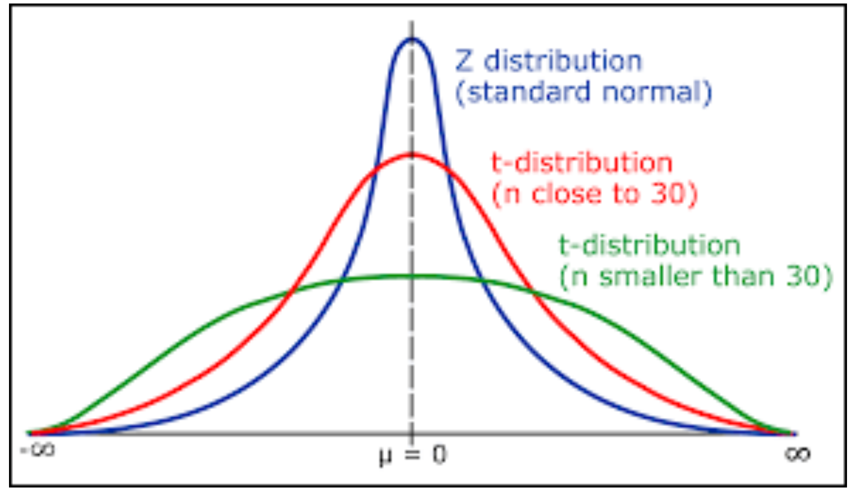
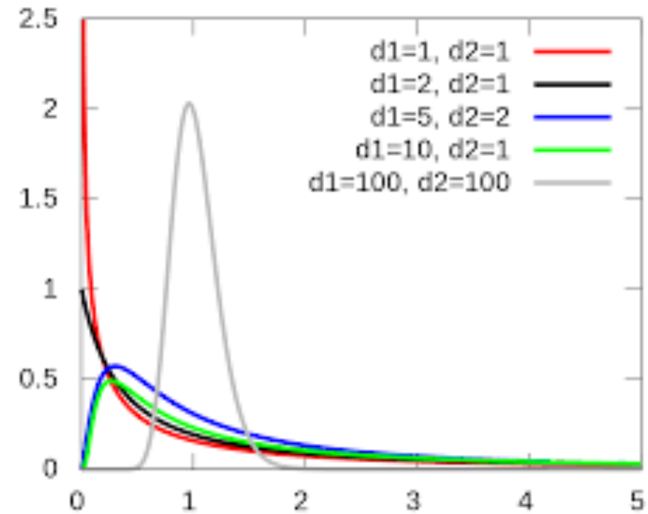
Revision: 10th Feb., 2021

Statistics: “t” Distribution



- Given *Z* as a standard normal random variable, and χ_N^2 is a χ^2 random variable (N degrees of freedom) independent of *Z*, then the ratio random variable $Z/\sqrt{\chi_N^2}$ has a t distribution with N degrees of freedom.
- The t distribution is symmetrical, bell-shaped, has mean & skewness & kurtosis = 0. The variance, however, = N/N-2, which of course converges to 1 as N -> infinity.
- Used to make inferences for the population mean when sampling from a normal population. Also for inferences about population regression coefficients.

Statistics: "F" Distribution



- Given two χ^2 random variables χ_N^2 and χ_M^2 then the random ratio variable $(\chi_N^2/N)/(\chi_M^2/M)$ has an F distribution with N & M degrees of freedom. These are described, obviously as the numerator and denominator degrees of freedom, respectively. Like the χ^2, the F distribution is positive and skewed to the right.
- Again, inferences for multiple regression models, analysis of variance models and multivariate means make use of this distribution.